

# Hydrodynamic Consideration and Limitations in Submerged Rocket Firings

RICHARD J. LABOTZ\*

*Aerojet-General Corporation, Sacramento, Calif.*

When a rocket engine is started under water, the inertia of the water initially in and around the engine restricts the gas flow. In engines with short starting transients this can result in excessive internal pressure. This paper presents an approximate analysis that can be used to determine the minimum starting transient for a given allowable pressure. The water is assumed to form a conservative system, so that the work done on the water by the gas is equal to the sum of the kinetic and potential energies stored in the water. The engine is represented as a cylindrical combustion chamber with a converging-diverging conical nozzle. By use of continuity, the flow zones in and around the engine are coupled to produce a group of non-linear differential equations that are easily solved numerically. Approximate methods of solution are given for plug nozzles and clusters of conical nozzles. Examples are given for a 20,000-lb-thrust engine with a single nozzle, both fully and partially flooded with water, and for an  $80 \times 10^6$ -lb-thrust engine with single nozzle, a cluster of nozzles, and a plug nozzle.

## Nomenclature

$A_1$	= cross-sectional flow area in the cylindrical combustion chamber
$A_2$	= spherical flow area at the throat for (conical) source flow
$E_k, E_p$	= kinetic and potential energies, respectively
$g$	= gravitational constant, 32.2 ft/sec <sup>2</sup>
$P$	= pressure, psf
$V$	= volume, ft <sup>3</sup>
$W$	= net work done by the gas on the water in evacuating the portion of the flow field denoted by the subscript
$x, x_1, y;$	
$y_1, y_2, z;$	
$z_1, z_2, r_3;$	
$s$	= defined in Fig. 1
$Y, a$	= dummy variables
$\rho$	= water density, lb/ft <sup>3</sup>

## Subscripts

$h$	= denotes in ambient water
$g$	= denotes in gas bubble
$0$	= ambient condition at initial gas-water interface
$c$	= cylindrical portion of the nozzle
$cc$	= converging portion of the nozzle
$dc$	= diverging portion of the nozzle
$fr$	= flow readjustment zone
$sf$	= spherical flow

## Introduction

ALTHOUGH sea launching (with underwater ignition) of a rocket offers some distinct advantages over land launching, the engine starting transient causes concern. The combustion gases can begin to flow out of the combustion chamber only by displacing the water initially in and around the nozzle. For engines that start rapidly, this restriction of the gas flow by the water, which has high density and inertia, can result in high internal gas pressure. This problem has been treated by Rogers<sup>1</sup> in order to predict the ignition pressure transient in a submerged rocket engine with no initial flooding of the nozzle (the gas-water interface was at the

nozzle-exit plane). He postulated that upon ignition a spherical gas bubble, beginning with zero radius at the nozzle-exit plane, grew in an infinite body of incompressible liquid. For this model he predicted the maximum pressure experienced due to a step change in gas flow rate from zero to the steady-state value. The results were correlated with some experimental data. Rogers and Lindsay<sup>2</sup> extended Rogers' analysis by changing the mode of bubble growth to one in which the bubble is formed in the space swept by a constant-diameter disk moving axially out the nozzle-exit plane. The starting transient restrictions were relaxed somewhat by expressing the gas flow rate as a power function of time. In addition, they approximated the case in which the nozzle is initially partly flooded by considering the acceleration of slug of water in a cylinder of finite length.

The present analysis solves the problem of how short the starting transient can be for a specified maximum pressure. The flow-rate transient corresponding to this minimum-time start is also found. Although the equations are derived for engines that are completely flooded at ignition, they can be modified to provide approximate solutions for partially flooded engines. The results should be useful in the design of engines for water-launch applications or in the determination of the degree of chamber flooding permissible with various engines, as well as in determining if a performance penalty (relative to surface launching) is placed upon a propulsion unit by submerged firing.

## Flow Model for a Single Conical Nozzle

The flow model used for the water emerging from a single conical nozzle is shown in Fig. 1. The engine has a cylindrical combustion chamber, and the starting flow of the water is assumed to be axial in both the combustion chamber and the conical converging section of the nozzle. (Although this assumption leads to a local error of a few percent for the converging portion of the flow due to the neglect of the radial component of velocity, the effect on the over-all flow is negligible.) The flow in the diverging conical section is assumed to be a (truly conical) source flow. (The water in the small spherical segment between the nozzle throat and the beginning of the source flow is neglected; consideration of this water would result in an additional term in the final equation without making any perceptible contribution to the accuracy of the over-all solution.) The next step is to provide an approximate model for the transient flow pattern beyond the nozzle exit. If it is kept in mind that we are seeking a de-

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\* Technical Specialist, Thermal Technology Department, Engine Technology Division, Liquid Rocket Operations. Member AIAA.

scription of the starting transient within the rocket engine rather than a detailed description of the exterior flow field, then a satisfactory model for the flow beyond the nozzle exit can be obtained by using the following simple approach.

Shortly after ignition, the water flowing from the conical nozzle at low velocity may appear, from some small distance, to be similar to a point-source flow. If so, at this same distance an equivelocity surface through the flow will be essentially spherical, and the developing flow pattern might behave as though there were a variable strength point source just outside the nozzle exit. Thus, for a short while after ignition, a reasonable flow pattern might be one that matches a conical flow from the nozzle exit to a spherical flow a little farther out. The shaded "flow readjustment zone" in Fig. 1 provides a smooth transition from the conical flow to the spherical flow. It is fortunate that this model is reasonably valid shortly after ignition when the water velocities are low, because during this critical period the gas bubble is small and the system is "hard," so that large pressure changes can be triggered by seemingly small changes in the rocket's start up characteristics. Conversely, at later stages of the starting transient when the water velocities are larger and the point-source flow model breaks down, the need for accuracy is decreased, because the gas bubble is now large, and it cushions the effects of irregularities in the gas flow.

The flow readjustment zone is subdivided into six parts by spherical surfaces, which intersect the nozzle-exit periphery and have their centers on the nozzle centerline. The first of these surfaces represents the end of the diverging conical flow, and the last represents the beginning of the spherical flow. The zone is assumed to extend to a spherical surface of radius  $1.06 r_3$ , where  $r_3$  is the nozzle-exit radius. Each of the spherical surfaces is assumed to be an equivelocity surface, and the fluid volume bounded by two adjacent surfaces is assigned the average velocity of the two surfaces. Both the value used for the outer radius of the flow readjustment zone and the manner in which the equivelocity surfaces converge were selected for ease of calculation. (As will become apparent later, the final results are rather insensitive to these values.) The zone of spherical flow begins at the outer boundary of the flow readjustment zone and extends to infinity. In practice, surface and bottom effects will limit the extent of the spherical flow, but the presence of such real boundaries has a negligible effect on the over-all results unless the engine is much closer to them than it normally would be in an underwater firing.<sup>2</sup> It is assumed that the vehicle prevents water from flowing into the spherical angle subtended by the nozzle-exit plane relative to the center of the spherical flow. This assumption credits the vehicle with more of a restrictive action

than it actually has and hence tends to be conservative. Vehicle motion is ignored, because the total times involved are so short that no appreciable vehicle motion is possible.

Some idea of what actually occurs can be obtained from the sequence of pictures shown in Fig. 2, which shows the combustion gas bubble as it emerges from the nozzle. Although in many respects what is occurring here is considerably different from what is happening at the beginning of the transient, the general flow pattern is about the same. The bubble surface velocity in this sequence is about 135 fps.

### Derivation of Equations for a Single Conical Nozzle

The starting transient is completely described by four differential equations covering the motion of the water-gas interface 1) within the cylindrical-combustion chamber, 2) within the converging portion of the nozzle, 3) within the diverging source flow regime, and 4) crossing the outer boundary of the flow readjustment zone. Because of the similarity of the four derivations, only the one for the converging section is given. The water is considered to be an infinite body of inviscid incompressible fluid initially at rest in a state of zero potential energy. At any time after ignition, the work

Fig. 1 Flow model for conical nozzle.

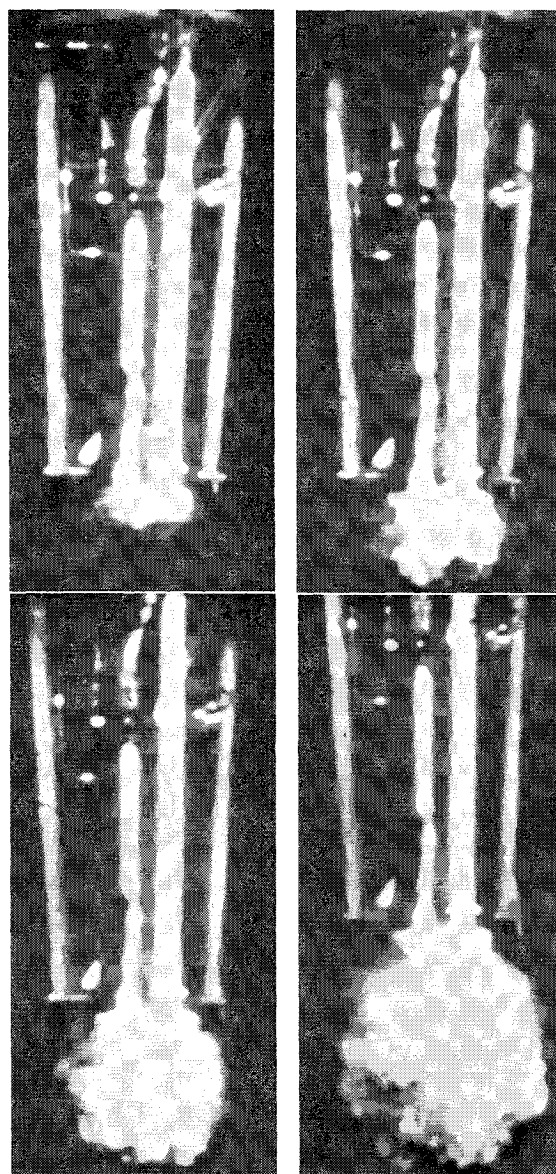
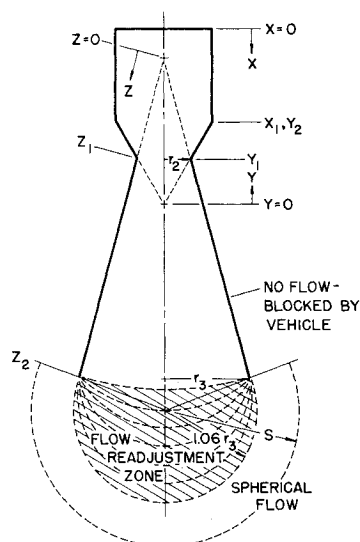


Fig. 2 Bubble formation during start transient.

done on the water by the gas is equal to the sum of the potential and kinetic energies of the water:

$$W = \int P_g dV = E_p + E_k \quad (1)$$

In general, the work integral can be evaluated only if the gas pressure is known as a function of the bubble volume. However, to obtain the important limiting case of the shortest possible starting time, a constant, maximum allowable gas pressure can be specified, so that the work term becomes simply the gas pressure multiplied by the instantaneous bubble volume. The potential energy is the work done in displacing the water against the hydrostatic back pressure and atmospheric pressure, hence

$$E_p = \int P_h dV \quad (2)$$

For small engines  $P_h$  can be assumed to be constant. The total work done at any point in the converging portion of the nozzle can be written as

$$W = P_g A_1 x_1 - \int_{y_2}^y P_g A_1 (a/y_2)^2 da \quad (3)$$

where  $a$  is a dummy variable. The first term represents the work done while the interface was in the cylindrical portion, and the second term gives the work done in the converging section, which is negative because the coordinate system used makes  $dy$  or  $da$  a negative quantity.

Combining Eqs. (1-3) can be shown to give

$$W - E_p = A_1 \left[ P_g - P_0 - \rho \frac{x_1}{2} \right] x_1 - A_1 \int_{y_2}^y \{ P_g - [P_0 + \rho x_1 + \rho(y_2 - a)] \} \left( \frac{a}{y_2} \right)^2 da \quad (4)$$

Making the substitution,  $P' \equiv P_g - P_0 - \rho x_1$ , and integrating yields

$$W - E_p = A_1 \left[ \left( P_g - P_0 - \rho \frac{x_1}{2} \right) x_1 + (P' - \rho y_2) \left( \frac{y_2^3 - y^3}{3y_2^2} \right) + \rho \left( \frac{y_2^4 - y^4}{4y_2^2} \right) \right] \quad (5)$$

The total kinetic energy of the water is the sum of the kinetic energy of the water in each of the flow areas. If the gas-water interface is moving with a velocity  $dy/dt$  in the converging section, then by use of continuity the axial velocity at any other point  $Y$  in the converging section can be shown to be

$$dY/dt = (y/Y)^2 dy/dt \quad (6)$$

The mass of water in an elemental volume of thickness  $dY$  is

$$dm_Y = (\rho/g)(Y/y_2)^2 A_1 dY \quad (7)$$

Equations (6) and (7) can now be combined to give the kinetic energy of the elemental volume of water, which is integrated to give  $E_{kcc}$ :

$$E_{kcc} = - \int_y^{y_1} E_{kY} = - \int_y^{y_1} \frac{\rho A_1}{2g} \left( \frac{dy}{dt} \right)^2 \frac{y^4}{y_2^2} \frac{dY}{Y^2} \quad (8)$$

or

$$E_{kcc} = \frac{\rho A_1 y^3}{2g} \left( \frac{y - y_1}{y_1 y_2^2} \right) \left( \frac{dy}{dt} \right)^2 \quad (9)$$

The kinetic energy of the water in the diverging section of the nozzle is obtained in much the same manner. The local velocity at  $z$  can be expressed as

$$dz/dt = (dy/dt)(A_1/A_2)(y/y_2)^2(z_1/z)^2 \quad (10)$$

and the local mass is given by

$$dm_z = (\rho A_2/g)(z/z_1)^2 dz \quad (11)$$

The kinetic energy of the water between  $z_1$  and  $z_2$  is

$$E_{kdc} = \frac{\rho A_1^2}{2g A_2} \left( \frac{dy}{dt} \right)^2 \left( \frac{y}{y_2} \right)^4 \left( \frac{z_2 - z_1}{z_2} \right) z_1 \quad (12)$$

In the flow readjustment zone, the volume and mass of the water in each subzone are calculated from simple geometric relationships. Application of the continuity equation gives the inlet and exit velocities for flow normal to the subzone boundaries. The kinetic energy for each subzone is obtained by using the average of the inlet and exit velocities. The sum of the kinetic energies for the six subzones is

$$E_{kfr} = \frac{1.45 \rho A_1^2}{2\pi^2 r_{3g}} \left( \frac{y}{y_2} \right)^4 \left( \frac{dy}{dt} \right)^2 \quad (13)$$

For the spherical flow region, the local velocity at radius  $s$  is

$$ds/dt = (A_1/4\pi K_0 s^2)(y/y_2) dy/dt \quad (14)$$

The mass of water in the shell of thickness  $ds$  is

$$dm_s = (4\pi K_0 \rho/g) s^2 ds \quad (15)$$

where  $K_0$  is the fraction of the spherical flow which is not blocked by the vehicle. Combining Eqs. (14) and (15) to give the local kinetic energy and integrating between 1.06  $r_3$  and  $\infty$  gives a total kinetic energy for the spherical flow

$$E_{ksf} = \frac{0.943 \rho A_1^2}{8\pi g K_0 r_3} \left( \frac{y}{y_2} \right)^4 \left( \frac{dy}{dt} \right)^2 \quad (16)$$

The equation governing the bubble growth and water motion when the gas-water interface is in the converging section can now be obtained by taking the difference between the total work done and the potential energy of the water as given in Eq. (5) and equating it to the sum of the kinetic energy terms as represented in Eqs. (9, 12, 13, and 16):

$$A_1 \left[ \left( P_g - P_0 - \frac{\rho x_1}{2} \right) x_1 + (P' - \rho y_2) \left( \frac{y_2^3 - y^3}{3y_2^2} \right) + \rho \left( \frac{y_2^4 - y^4}{4y_2^2} \right) \right] = \frac{\rho A_1}{2g} \left( \frac{dy}{dt} \right)^2 \left\{ y^3 \left( \frac{y - y_1}{y_1 y_2^2} \right) + \left( \frac{y}{y_2} \right)^4 \left[ z_1 \left( \frac{A_1}{A_2} \right) \left( \frac{z_2 - z_1}{z_2} \right) + \frac{1.15 A_1}{\pi^2 r_3} \left( \frac{y}{y_2} \right)^2 + \frac{0.943 A_1}{4\pi K_0 r_3} \right] \right\} \quad (17)$$

Equation (17) is then solved algebraically for the velocity of the gas-water interface  $dy/dt$ . Using a similar approach, the equations governing the interface motion in the cylindrical portion and the diverging portion were found to be

$$\frac{dx}{dt} = \left[ 2gx \left( P_g - P_0 - \frac{\rho x}{2} \right) \right]^{1/2} \div \left\{ \rho \left[ (x_1 - x) + y_2 \left( \frac{y_2 - y_1}{y_1} \right) + z_1 \left( \frac{A_1}{A_2} \right) \left( \frac{z_2 - z_1}{z_2} \right) + \frac{1.45 A_1}{\pi^2 r_3} + \frac{0.943 A_1}{4\pi K_0 r_3} \right] \right\}^{1/2} \quad (18)$$

$$\frac{dz}{dt} = (2g)^{1/2} \left\{ W_c + W_{cc} + \frac{A_2}{z_1^2} \left[ \frac{P''}{3} (z^3 - z_1^3) - \frac{\rho}{4} (z^4 - z_1^4) \right] \right\}^{1/2} \div \left\{ \rho A_2 \left[ z^3 \left( \frac{z_2 - z}{z_1^2 z} \right) + \frac{1.45 A_2}{\pi^2 r_3} \left( \frac{z}{z_1} \right)^4 + \frac{0.943 A_2}{4\pi K_0 r_3} \left( \frac{z}{z_1} \right)^4 \right] \right\}^{1/2} \quad (19)$$

where the work done in evacuating the cylindrical section is

$$W_c = A_1 x_1 (P_g - P_0 + \rho x_1/2) \quad (20)$$

and the work done in evacuating the converging section is

$$W_{cc} = A_1 \left[ (P'' - \rho y_2) \left( \frac{y_2^3 - y_1^3}{3y_2^2} \right) + \rho \left( \frac{y_2^4 - y_1^4}{4y_2^2} \right) \right] \quad (21)$$

and

$$P'' \equiv P_g - P_0 - \rho(x_1 + y_2 - y_1 - z_1) \quad (22)$$

The velocity of the interface as it passes out of the readjustment zone is given by

$$\frac{dR}{dt} = \left[ \frac{(W_c + W_{cc} + W_{dc} + W_{fr})g}{2\pi\rho K_0(1.06r_3)^3} \right]^{1/2} \quad (23)$$

$W_{dc}$  and  $W_{fr}$  for the diverging cone and flow readjustment zones are given by

$$W_{dc} = \left[ \frac{P''}{3} (z_2^3 - z_1^3) - \frac{\rho}{4} (z_2^4 - z_1^4) \right] \frac{A_2}{z_1^2} \quad (24)$$

and

$$W_{fr} = 3.69[P_0 + \rho(x_1 + y_2 - y_1 + z_2 - z_1 + \frac{2}{3}r_3)]r_3^2 \quad (25)$$

Equations (17-19) are fairly complex nonlinear differential equations with no apparent analytical solutions. However, they are rather easily integrated numerically as they are explicit in the velocity term. Doing this, one obtains both the displacement and velocity histories of the gas-water interface as it moves down the nozzle. These histories are converted into gas flow rate starting transient data by calculating the rate of increase in volume of the gas bubble (the product of interface velocity and local flow area) as a function of time and then calculating the gas flow rate transient required to feed this expanding bubble. The latter calculation can be rather simple or very complex, depending upon the degree of accuracy required; in most cases the following simple approach will provide 90% of the answer with 10% of the effort required for a more sophisticated analysis.

Let us assume that all of the gas in the bubble is at a uniform temperature and pressure. Thus the product of the rate of increase in bubble volume and the density of gas will give the mass flow transient. Just as in the argument for the "flow readjustment zone" model, this assumption should be reasonably valid for the critical period immediately after ignition during which a small error in the mass flow transient could produce excessive pressure, because at this time the gas bubble should be a nearly homogeneous mass. Later in the starting process the assumption of uniform gas density still represents a good approximation. If the maximum allowable pressure is set equal to (or greater than) the steady-state chamber pressure, then the gas flow within the bubble will be subsonic throughout the whole starting process. Only in the vicinity of the throat is the density of the gas somewhat lower than the stagnation value, but the volume occupied by this higher velocity gas is only a small portion of the total gas volume, so that the average density of the gas in the bubble will remain close to the stagnation value. Thus, assuming that all of the gas in the bubble is at the stagnation density introduces only a slight error.

To obtain exact results for the situation in which the engine is only partially flooded requires a considerably more complex solution, because it is impossible to achieve the assumed step increase in bubble pressure at ignition when a gas bubble is already present in the chamber. To obtain the exact solution would require changing the work term to account for a variable bubble pressure. However, the simple solution presented here can still supply a reasonable approximation by assuming that the initial interface is the injector or grain end and that the chamber is flooded up to this hypothetical injector. The starting transient obtained in this manner will not be the shortest possible nondamaging transient, but it will be a conservative approximation of it.

Early in the analysis it was assumed that the maximum gas pressure existing in the bubble would always be acting on the water at the gas-water interface, and  $P_g$  was set equal to the maximum allowable pressure. Although this assumption obviously is valid shortly after ignition, it may seem suspect

as the mass flow rate and velocities continue to increase. Actually this assumption should be valid for the whole starting transient. As pointed out earlier, the gas-water interface moves relatively slowly, so that the local static pressure of the gas is nearly equal to its stagnation pressure. Furthermore, the nozzle will act as a fairly efficient venturi for the subsonic transient flow, so that there will be little loss in stagnation pressure of the gas flowing through it. Thus, the pressure at the interface will be only slightly less than chamber pressure for a considerable portion of the transient, until nozzle choking (which represents, in the practical sense, the end of the starting transient) is closely approached. At this time, the mass flow rate will be at least equal to its steady-state value, because the chamber pressure has been set equal to its maximum allowable value as the critical transient criterion.

### Clusters of Conical Nozzles and Plug Nozzles

The treatment of a cluster of conical nozzles that are initially flooded is only slightly different from that of a single nozzle. The easiest approach is to take the two limiting cases between which the real case will lie. In one limiting case, each nozzle acts as though the others are not there (i.e., the single nozzle analysis), and in the other, it is assumed that all of the nozzles feed common flow readjustment and spherical flow zones. The flow readjustment zone is given a base diameter equal to the diameter of the cluster. The total kinetic energy for the combined readjustment and spherical flow zones is then divided by the number of nozzles present, and this fraction of the total is used in the equations developed for the single nozzle. The real behavior of the cluster should lie somewhere between these two extremes; the external flow interaction will be greater than that for the first case but less than that for the second.

For a plug nozzle, the flow leaving the nozzle approximates a toroidal, diverging flow from a line source. Although no equations for plug nozzles are included in this paper, some numerical results obtained from a flow model similar to the one just described are included for the sake of comparison.

### Numerical Results

Several numerical examples have been solved using the assumption of a constant gas density within the bubble, corresponding to a maximum allowable bubble pressure equal to the steady-state chamber pressure. The first two examples are for a 20,000-lb-thrust, 310-psia engine with a 17-in.-long, 10½-in.-diam combustion chamber and a nozzle having an 18° half-angle converging section, a 7.5-in.-diam throat, a 13.5° half-angle exit cone, and an exit diameter of 16.5 in. The gas density was assumed to be 0.125 lb/ft³. These two examples differ only in that in one case the engine was initially completely flooded, whereas in the other it was flooded only to the throat. The engines were assumed to be exhausting 35 ft below the surface while maintaining the 310-psia chamber pressure. Figure 3 shows that the steady-state flow rate is attained in ~30 msec with the completely flooded engine and in ~15 msec with the partially flooded engine; however, since the partially flooded engine contains an initial gas bubble, the 15 msec is a conservative figure, and the difference between the two cases will be somewhat greater than these time values indicate.

The relative magnitude of what were originally the kinetic energy terms in Eq. (19) reflect the amounts of kinetic energy being put into the various portions of the flow field and hence indicate the relative importance of the zones in restricting the initial flow of gas. For the completely flooded engine, the relative values at ignition are 0.38 for the cylindrical zone, 0.16 for the converging zone, 0.39 for the diverging zone, 0.04 for the readjustment zone, and 0.03 for the spherical zone. When the interface or bubble surface is

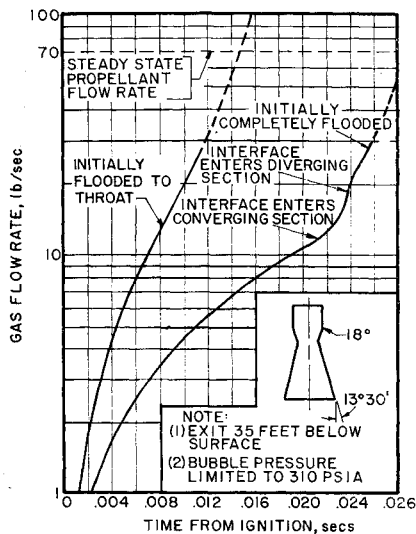


Fig. 3 Start transient for 20,000-lb-thrust engine.

at the throat, the relative values are 0.86, 0.08, and 0.06 for the diverging, readjustment, and spherical zones, respectively. When the bubble has passed through the nozzle, the relative values for the readjustment and spherical zones are 0.54 and 0.46. These values indicate that until the bubble has passed through the nozzle throat, the flow readjustment zone and spherical zone have very little influence. During the critical early portion of the firing, one could almost completely ignore both of these zones and still not be much in error. Thus, small inaccuracies in the models for these zones have an almost negligible effect on the accuracy of results for the portions of the transient that are of interest. Refinements in this part of the model at the expense of additional complexity would be hard to justify.

The third example is a comparison of approximate starting transients for various nozzle configurations for the Sea Dragon vehicle, an 80,000,000-lb-thrust liquid-propellant, sea launched booster. The transients for the plug nozzle, eight and four nozzle clusters, and a single conical nozzle are shown in Fig. 4. In each case it is assumed that the nozzle is initially flooded to the throat. The clustered nozzles and single nozzle are geometrically similar. The differences in nozzle exit depths approximately reflect the way the vehicle floats with each nozzle. These exit depth variations are reflected to a certain extent in the starting transients, due to corresponding variation in the hydrostatic pressure that the bubble must overcome, but the same bubble pressure is assumed for all configurations. The mass flow transients for the clusters and the plug nozzle are shown as bands, which were obtained by taking two limiting flow models for each case and evaluating the equations obtained with these models; the actual transients should lie within the bands.

It is not too surprising that the plug nozzle has the shortest starting transient among the configurations studied; with a line source of flow and an essentially toroidal bubble, it has a large unrestricted surface area over which bubble growth can take place. Relative to partially flooded conical nozzles, the plug nozzle would experience only a minimal amount of restrictive action from the water and hence would have a short starting transient.

The difference in transient times between the single nozzle and a cluster is primarily a function of nozzle size. An examination of Eqs. (18-20) will indicate that geometrically similar nozzles with the same net driving pressures will have

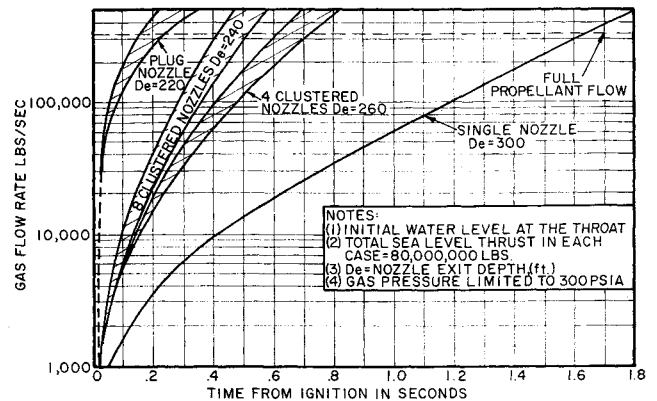


Fig. 4 Sea Dragon nozzles.

the same interface velocities at corresponding points in the nozzle. From this it is apparent that both the net inertial resistance and the transient time vary essentially linearly with individual nozzle size or with  $(\text{thrust}/\text{nozzle})^{1/2}$ , all other factors remaining constant. A cluster with a lower thrust/nozzle experiences a smaller inertial effect than a single nozzle with the same total thrust. A physical explanation is that the mass of water to be accelerated is proportional to the volume of the nozzle or some characteristic length cubed. The force tending to drive out the water is proportional to the area over which the gas pressure is acting or some characteristic length squared. The net size effect would then show the inertial resistance increasing approximately linearly with nozzle size. The fact that the starting transients for the various Sea Dragon nozzles do not quite follow this linear relationship is primarily due to the different hydrostatic back pressures.

Before leaving the examples it should be pointed out that the gas pressures used in these calculations were fairly low. It can be shown that for a flooded engine of a given thrust the minimum transient time is roughly inversely proportional to the chamber pressure. Thus, had the chamber pressures in the examples been double the values used (with nozzle dimensions decreased sufficiently to hold the thrust constant), the minimum transient times obtained would have been approximately halved.

## Conclusions

Two principal conclusions can be drawn from the analysis and results presented here:

- 1) The minimum transient times calculated are comparable to those for some real engines. Thus in selecting a propulsion unit for an underwater ignition operation, the starting transient of the unit should be checked to determine whether excessive gas pressure may be encountered.
- 2) A fairly heavy premium is paid for having the engine completely flooded. This penalty can be circumvented either by placing a diaphragm over the nozzle exit or by maintaining an initial gas bubble within the engine.

## References

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- <sup>2</sup> Rogers, K. W. and Lindsay, A. I., "Study of underwater rocket firing," Univ. of Southern California Engineering Center Rept. 100-101 (February 1964).